## Report

(1) Name of Lecturer: Jin Feng

(2) Title of Lecturer: Associate Professor

(3) Affiliation: Department of Mathematics, The University of Kansas, USA

(4) Short Biography: Jin Feng is an associate professor in Mathematics Department at the University of Kansas. His research focuses on probability and related PDE analysis, particularly those with a variational/optimal control nature. His recent works span areas of probabilistic large deviation theory, viscosity solution for Hamilton-Jacobi equations and optimal control, optimal mass transportation, and nonlinear analysis. Recently, he was awarded the Keeler intra-university professorship and is spending his fall semester of 2011 to study, with colleagues in Aerospace engineering department at University of Kansas, a few mathematical issues arising from fluid mechanics.

His past and current research has been supported by US Army Research Office, the National Science Foundation and American Institute of Mathematics.

(5) Subject and Schedule of the Lectures:

As a part of the course ``Modern Probability'' in Department of System Cybernetics, Graduate School of Engineering

**January 10, 2012**: 15:30-16:30 Optimal controlled PDE and Hamilton-Jacobi equations in space of probability measures I; 16:30-17:00 Free Discussion

**January 11, 2012**:  $\underline{10:00-11:00}$  Optimal controlled PDE and Hamilton-Jacobi equations in space of probability measures II;  $\underline{11:20-12:20}$  Optimal controlled PDE and Hamilton-Jacobi equations in space of probability measures III;  $\underline{13:00-14:00}$  Free Discussion

**January 12, 2012**:  $\underline{10:00-11:00}$  Optimal controlled PDE and Hamilton-Jacobi equations in space of probability measures IV;  $\underline{11:20-12:20}$  Optimal controlled PDE and Hamilton-Jacobi equations in space of probability measures V;  $\underline{13:00-14:00}$  Free Discussion

In this series of lectures, I will first introduce two classes of first order Hamilton-Jacobi-Bellman (HJB) equations in the space of probability measures, and then develop a well posed-ness theory for both the Cauchy and resolvent type problems.

In first lecture, I will derive two Hamiltonians using probabilistic large deviation theory applied to mean-field models from statistical mechanics, provide their connections with optimal controlled PDEs in space of measures, and explain some a priori estimates at microscopic level using probability theory.

In the second lecture, I will introduce necessary background about optimal mass transportation theory and use the language of such theory to prove uniqueness (comparison principle) result for HJB equations involving one of the Hamiltonians (the nicer one).

In the third lecture, I will improve the comparison principle result in the last lecture by regularization method and obtain continuity result of the solution under a reasonably weak topology (the Wasserstein-2 metric).

In the fourth lecture, I will introduce new ingredients to show the uniqueness (comparison principle) of HJB equations for the second Hamiltonian which has a strong singular term. I will also explain the relation of such equation to the Onsager-Joyce-Montgomery theory of 2-D turbulent vortex flows.

In the fifth lecture, I will explain a constructive existence theory by using dynamical programming for controlled PDEs in space of probability measures. In doing so, we will connect one of the Hamiltonians with variational problem giving compressible Euler equation for potential flows. A class of action functional defined over space of probability-measure-valued-curves will play a key role in this topic.

## (6) Comments:

