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Entrance Examination for Master's Course, 2012 April Enrollment

Department of Quantum Matter

Graduate School of Advanced Sciences of Matter

Hiroshima University

MAJOR SUBJECTS
(Engineering Field Problems)

August 22, 2011, 13:00～16:00

Notices

(1) This booklet includes the following sheets.

Problem sheets (including the cover)	6 pages
Answer sheets	3 pages
Memo sheet	1 page

(2) There are five problems [1]～[5], the categories of which are indicated in the boxes ☐ .

(3) Solve three problems out of these five problems.

(4) One answer sheet should be used for one problem. Write the problem number and its category name at the upper left corner of the answer sheets. The backside of the sheets can be used.

(5) Write your identification number on all the sheets.

(6) Return all received materials together with the answer sheets in one set.

(7) Write the numbers of the answered problems in the boxes below.

Entrance Examination for April 2012 Enrollment
Graduate School of Advanced Sciences of Matter, Hiroshima University

Department of Quantum Matter (Engineering Field)

[1] Electromagnetism

1. Consider a solid conducting sphere A with radius a inside a concentric hollow conducting sphere B with inner radius b (Fig. 1). The center of the spheres is the origin O. The electric potential of A is fixed at a positive value and that of B is 0. The space between A and B is vacuum. The dielectric constant of vacuum is ϵ_0. r is the radial coordinate in the spherical polar coordinate system.

   (1) Give the direction and the magnitude of the electric field $\boldsymbol{E}(r)$ and the electric potential $V(r)$ at a point between A and B, if the surface charge density on the surface of A is σ.

   (2) Draw a graph of the magnitude of the electric field $|\boldsymbol{E}(r)|$ for $0 \leq r \leq b$ as a function of r.

   (3) Give σ, when the electric potential of A is V_1. Write down the electric field $\boldsymbol{E}(r)$ and the electric potential $V(r)$ which are obtained from this specific value of V_1.

   (4) Give the capacitance C for A and B.

   (5) Show that the electric potential $V(r)$ at a point between A and B satisfies Laplace's equation. Since the system is spherically symmetric, the Laplacian in the spherical polar coordinate system can be expressed as $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r})$.

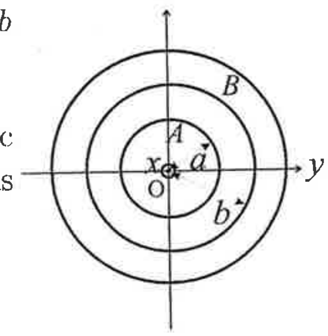


Fig. 1

2. Consider a cylindrical conductor placed on the z axis (Fig. 2). Its radius is a and its length is l. A current with a current density $\boldsymbol{j} = j\boldsymbol{e}_z$ is flowing uniformly inside the conductor. l is sufficiently long and the effect of both ends can be neglected. The magnetic permeability of vacuum is μ_0.

   (1) Give the magnitudes and the directions of the magnetic field $\boldsymbol{H}_i(r)$ inside ($r < a$) of the conductor and the field $\boldsymbol{H}_o(r)$ outside ($r > a$) of the conductor. r is the distance from the z axis.

   Another conductor having the same diameter and length as the first one is placed parallel with the first one to make a circuit as shown in Fig. 3. The distance between them is $D(\gg a)$. The magnetic field inside of the conductors and the magnetic field of the connecting wires can be neglected.

   (2) Give the magnetic flux Φ through the circuit consisting of these conductors.

   (3) Give the self inductance L of the circuit.

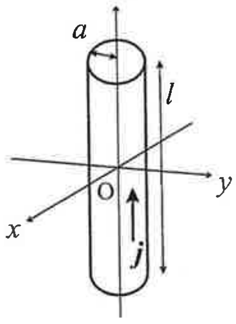


Fig. 2

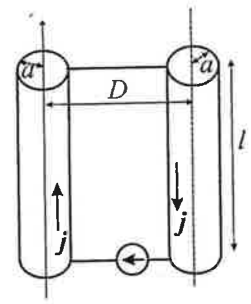
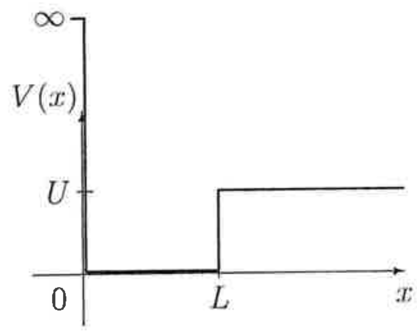


Fig. 3

[2] Quantum Mechanics

Consider the ground state of a particle of mass m in a potential $V(x)$ that is infinite at $x < 0$, zero for $0 < x < L$, and has a constant value of U for $x > L$,

$$V(x) = \begin{cases} +\infty & \text{for} \quad x < 0 \\ 0 & \text{for} \quad 0 < x < L \\ U & \text{for} \quad x > L \end{cases}$$



The ground state energy E has a value between zero and the potential U at $x > L$.

1. Write down the time independent Schroedinger equation for an energy eigenstate $\psi(x)$ with energy eigenvalue E of the particle of mass m in the potential $V(x)$.
2. Give the boundary condition at $x = 0$.
3. Find a wavefunction $\psi_1(x)$ that solves the Schroedinger equation for $E > 0$ between $x = 0$ and $x = L$ and satisfies the boundary condition at $x = 0$.
4. Give the boundary condition for $x \to \infty$ if the energy is $E < U$.
5. Find a wavefunction $\psi_2(x)$ that solves the Schroedinger equation for $x > L$ and $E < U$ and satisfies the boundary condition for $x \to \infty$.
6. Give the boundary conditions at $x = L$.
7. For a specific value of $U = U_0$, the energy of the lowest energy eigenstate is found at $E = U_0/2$. Use the boundary conditions at $x = L$ to show that the potential U_0 is given by
$$U_0 = \frac{9}{16}\frac{\pi^2\hbar^2}{mL^2}.$$

[3] Semiconductor Engineering

Consider the drift motion of electrons in a semiconductor. Under an electric field F, the equation of motion for the drift velocity $v(t)$ at time t can be written as

$$\frac{dv(t)}{dt} = -\frac{eF}{m^*} - \frac{v(t)}{\tau} \qquad \text{(a)}.$$

Here, e and m^* are the elementary charge and the effective mass of the electron, respectively, and τ is a constant. Let F be a non-zero constant and the initial velocity be $v(0) = 0$.

1. Find $v(t)$ for $t \geq 0$ by solving equation (a).
2. For large t, $v(t)$ approaches an asymptotic value v_{sat}. Express v_{sat} using the given symbols.
3. Draw a graph of $v(t)$ for $t \geq 0$. In the graph, indicate the time $t = \tau$ on the t axis.
4. Express the electron mobility μ using the given symbols.
5. Give the relation between the electrical conductivity σ and the mobility μ in a semiconductor with an electron density of n.
6. Explain the physical meaning of the constant τ.

Quantum mechanically, the wave nature of electrons must be considered. Let us assume the following relation between the wavenumber k and the eigenenergy E of electrons;

$$E = E_0 + \frac{\hbar^2}{2m^*}(k - k_0)^2 \qquad \text{(b)}.$$

Here, $\hbar = h/(2\pi)$ with h being the Planck constant and E_0 and k_0 are a constant of energy and a constant of wavenumber, respectively.

7. Give the relation between E and the angular frequency ω of the electron wave.
8. The particle-like motion described by equation (a) can be interpreted as the motion of the wavepacket of the electron. In general, the velocity of a wavepacket with a central wavenumber k_C is given by $v = d\omega/dk|_{k=k_C}$ based on the relation between the angular frequency ω and the wavenumber k. For the electron obeying equation (b), derive the relation between the central wavenumber k_C and the velocity v of the wavepacket. Draw a graph of v as a function of k_C.
9. Using the equation $dk_C/dt = G/\hbar$ for the central wavenumber k_C of a wavepacket under an external force G, derive the first term in the right hand side of equation (a).

[4] Thermodynamics and Statistical Mechanics

1. Explain briefly the canonical distribution and the partition function, and describe the general numerical expressions for them. Use all the keywords given below, and give the definitions of the symbols used.
   Keywords: thermal bath, probability, normalization, microstate, temperature, energy

2. Consider a system of N atoms in a line. Each atom can have two quantum states, State 1 and State 2, whose energies are
$$\epsilon_1 = -\epsilon, \quad \epsilon_2 = +\epsilon$$
   respectively, where $\epsilon > 0$. We assume that the interaction between atoms can be neglected.

   (1) Find the number of cases in which the number of atoms in State 1 is N_1.

   (2) For the case described in (1), give the entropy as a function of N_1 and find it's maximum value. Assume $N \gg 1$, $N_1 \gg 1$ and $N - N_1 \gg 1$, and use the Stirling formula,
$$\ln n! \simeq n(\ln n - 1) \quad (n \gg 1).$$

3. Consider the case when the system described in 2. is in thermal equilibrium with a heat bath of absolute temperature T.

   (1) Find the number N_1 of atoms in State 1.

   (2) Find the values of N_1 and the entropy in the low temperature limit, $T \to 0$, and in the high temperature limit, $T \to \infty$.

   (3) Explain the results obtained in (2) in terms of the minimization of free energy, based on the relation between the Helmholtz free energy, the energy and the entropy.

[5] Materials Science

1. Solve the following questions concerning the Langmuir isotherm with respect to gas molecule adsorption on a solid surface.

(1) Write down the rate equation for $\frac{d\theta}{dt}$, where θ is the surface coverage. The rate constants of adsorption and desorption are k_a and k_d, and the gaseous pressure is p.

(2) Write down the equilibrium condition and show that the surface coverage $\theta = \frac{K p}{1+K p}$ at the equilibrium condition. Here, K is defined as $K = \frac{k_a}{k_d}$.

(3) The table shows data for a gas adsorption on a material surface at a constant temperature. Show that the data satisfy the Langmuir isotherm. Obtain the K value, which was defined in question (2), and the maximum adsorption amount in suitable units. Here, the volume $V(p)$ gives the value at the standard conditions.

p/hPa	200	400	600
$V(p)$ /cm^3	10	16	20

2. Answer the following questions concerning the battery cell reaction,

$$A^{2+} + B \rightarrow A + B^{2+}.$$

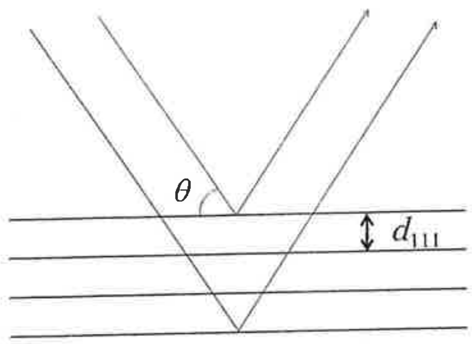
(1) Give the standard cell potential of this cell reaction. Here, the standard Gibbs energy of the reaction is ΔG^0, and the Faraday constant is F.

(2) Give the logarithmic value of the equilibrium constant if the standard cell potential is +1.2V at a temperature of 300K. Here, for simplicity, the Faraday constant and the gas constant are considered to be $F = 1 \times 10^5$[C/mol] and R = 8[J/molK], respectively.

(3) Explain the nature of the equilibrium state at the condition in question (2).

3. Answer the following questions concerning diffraction techniques.

(1) As shown in the figure, a diffraction of X rays with wave length λ is observed at the glancing angle θ, which corresponds to the diffraction from the (111) plane of a cubic crystal. Express the side length a of the unit cell using λ and θ.



(2) The C-H bond length of Sucrose was determined to be 0.096 nm by X ray diffraction and determined to be 0.110 nm by Neutron diffraction. Explain the reason why such a difference was obtained in these techniques.